

THE
ELEMENTS
OF
GAUGING:
OR,

A Solution of all the Necessary
PROBLEMS in Gauging;

Together with the
REASONS of Them
Geometrically Demonstrated:

As a SUPPLEMENT to Mr. *Ward's*
Gauger's Practice.

By *James Lightbody*, P. M.

L O N D O N,

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THE ELEMENTS

OF GAUCING

A TREATISE ON THE ART OF
GAUCING



OF THE

Geometrical Description

OF THE SUPPLEMENT TO THE

BY J. M.

T H E
E L E M E N T S
O F
Gauging, &c.

GAUGING consists chiefly in the Mensuration of Squares, Circles, and Ovals, the Foundation or Basis whereof is Geometry, and it is very necessary that he who aims at Gauging should have a competent share of Knowledge therein, without which a Man may be a good Officer, but can never be a good Gauger; for unless he have the Reasons of every thing he does, he can never have a true Notion of what he is about: Moreover, the reasonable part being well digested, has such Weight in it self, that nothing can root it out of a Man's Thoughts; and upon any occasion, he is not only able to work any Question put to him, more readily than without it, but to discourse the same with greater Judgment than he who wants so necessary an Accomplishment.

A 2

of

Of a Geometrical Square.

A Geometrical Square is a Figure produced from the Multiplication of two right Lines one into the other, as if the two right Lines *A B* and *A D* be multiplied one into the other, thereof is made the Quadrangle *A B C D*. If then *A B* be five Foot and *A D* eight, the whole Quadrangle shall be 40 square Feet, as appeareth by the prick'd Lines in the Diagram.



Now we shall suppose this to be a Back or Cooler, whose longest side is 173.2 Inches, and shortest 124.5 Inches, the multiplying of these two Numbers the one into the other, that is to say, *A B* into *A D*, the Product will be the Area in superficial Inches and parts.

Then to find its Content in Ale or Wine Gallons, you must find the Area in Gallons, that is, how many Gallons it holds upon one Inch deep in Liquor, by dividing the Sum of superficial Inches by the number of solid Inches contained in an Ale or Wine Gallon, viz. 282, or 231.

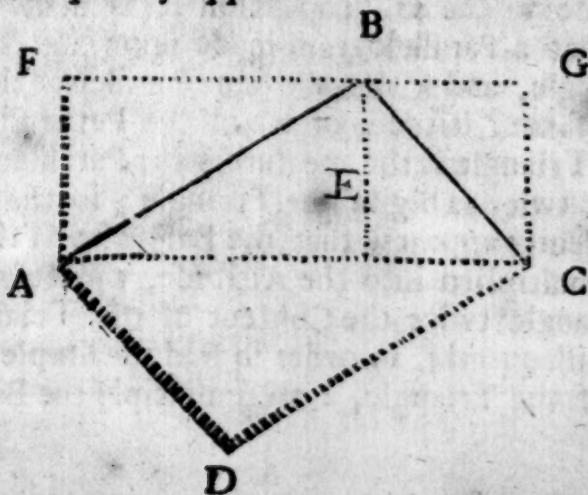
Quest.

Quest. What is the Reason that the dividing by 282 or 231 produces the Area in Ale or Wine Gallons?

Ans. 282 and 231 are the number of Inches contained in an Ale or Wine Gallon, according to the Statute: Therefore when you have found how many Inches in Superficie any Cooler contains, and knowing that 282 square or solid Inches is in a Gallon of Beer or Ale, you are obliged by the common Rules of Arithmetick, to know how often 282 is in the number of Inches found; which shews you how many Gallons are contained in the whole number of superficial Inches. For 282, the number of Inches in a Gallon of Ale, may be compared to 20, the number of Shillings in a Pound; for if you have 3567 Shillings, you must divide that number by 20, to know how many Pounds is therein.

Of TRIANGLES.

LET a Triangle be right Angled, Oblique, or Acute, I say, which soever of these it be, it is always equal to the half of a Square or Oblong, having two sides equal to the Base, and Perpendicular of the same Triangle, as by the following Figure will plainly appear.



Let

Let ABC be a Triangle, whose Base AC is 460, and Perpendicular E is 370, the Content in superficial Inches of this Triangle is 170200, which is equal to the half of $ABCD$, or $FGCA$; for if the Triangle AFB be 100000, BGC will be 70200.

Therefore, since you multiply the one side of a Quadrangle or Square by the other, to find the superficial Content, you are by Reason obliged to multiply the half of the one side by the whole of the other, to find the superficial Content of a Triangle, it being the half of a Quadrangle.

The demonstrative Account of this Doctrine is grounded upon the 34. and 41. Proposition of the first Book of *Euclid*. In the first it is taught and demonstrated, that the Diagonal of a Parallelogram divides the whole into two equal Triangles, and consequently that the whole Parallelogram is equal to two Triangles, contained betwixt the parallel sides and the Diagonal: And this holds not only as to Rectangular Squares, but as to all Parallelograms.

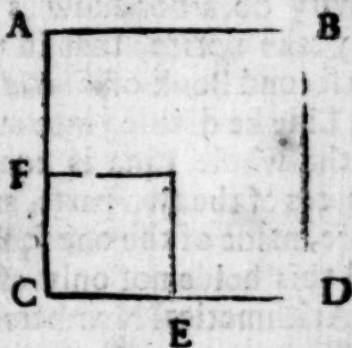
Besides in the 41. Proposition it is demonstrated, that a Parallelogram made upon the Basis of a Triangle, and within the same Parallels, (that is, of the same Altitude, of which the Perpendicular of the Triangle is the measure,) the Parallelogram shall be twice as big as the Triangle; so that here it evidently appears, that the Base of a Triangle being multiplied into the Altitude, the Result is a Quadrangle twice the Content of the Triangle; and consequently, in order to find the simple Content of the Triangle, I must multiply the Base by
the

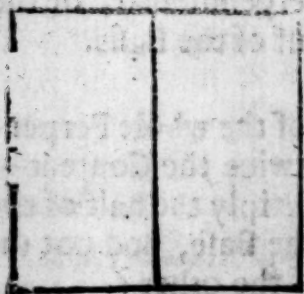
the half of the Altitude or Perpendicular, or the whole Perpendicular by the half of the Basis.

Quest. Since a Square made of the whole Perpendicular and the whole Base, is twice the Content of the Triangle, why do not I multiply the half of the Perpendicular by the half of the Base, and not the whole of the one by the half of the other?

Answ. If I multiplied only the half of the one by the half of the other, instead of finding the half Content of the Square, which is equal to that of the Triangle, I should find but a fourth part of it, as by the following Figure appears.

The Reason of this is, because the Proportion betwixt two Squares is not the same as betwixt two Roots: As for Example, tho' 4 be the half of 8, yet the Square of the first, viz. 16, is but the fourth part of the Square of 8, viz. 64. In like manner, tho' 10 Foot be the half of 20, yet a Superficie of 10 Foot square is but the fourth part of another of 20 Foot square, as suppose AD to be twenty Foot square, take off 10 Foot of the Base CD, which is CE, and 10 Foot of the Altitude CF, this squared, makes but the fourth part of the whole ABCD, so that it is evident, that to find out the half of a Square, I must multiply the whole of one side by the half of the other:





other: As for Example, the Parallelogram described upon the whole side, and half of the side, makes a Quadrangle equal to the half of the whole Square; as by the annexed Figure doth appear: From all which we may easily con-

clude, that since a Quadrangle made upon the Basis and Altitude of a Triangle, is the double of the Triangle, then the simple Content of the Triangle is represented by a Quadrangle, made upon the whole Altitude, and half of the Base; and *Vice Versa*, or (which is the same thing) multiplying the whole Altitude or Perpendicular by half the Base, and *è contra*, you have the superficial Content of the Triangle.

The perfecter Account of this Proportion betwixt a Square and its Root, is only drawn from the Doctrine of Proportions, which tho' it be highly pleasant and satisfactory in it self, yet our limited brevity does not allow us here to insert. We shall only take notice, that in the fourth Proposition of the second Book of *Euclid*, it is demonstrated, that if a Line be divided into any two parts, the Square of the whole Line is equal to the two separate Squares of the two parts; and besides, to two Squares more, made of the one squared by the other twice; and this holds not only of Geometrical Lines, but of Arithmetical Numbers.

As for Example, 7 is an entire Number, of which the Square is 49, suppose the Root 7 be divided

of Gauging.

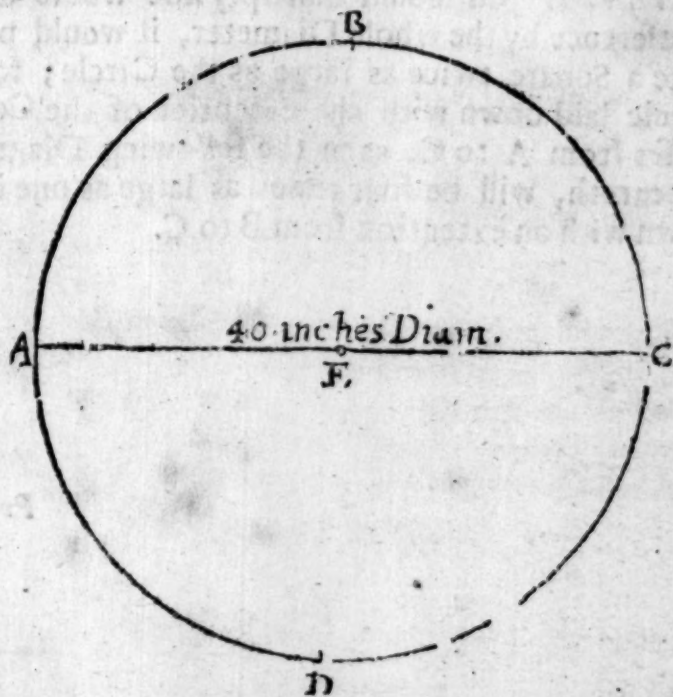
9

divided into two parts, 3 and 4, the Square of 3 is 9, and the Square of 4 is 16, the Sum whereof is 25, which is far short of the Square of 7, viz. 49, so that to make it up the one Number must be multiplied into the other twice, viz. 3 into 4, which makes twice 12, or 24, now 24 being added to the first Square 25, makes up 49.

Note, That multiplying the Perpendicular into the Base is the same thing as making a Quadrangle of the Perpendicular and Base.

Of a Circle.

A Circle is chiefly applied to Cask-Gauging, but before I apply it I shall first shew you why the Content of it is found by the Rule directed.



B

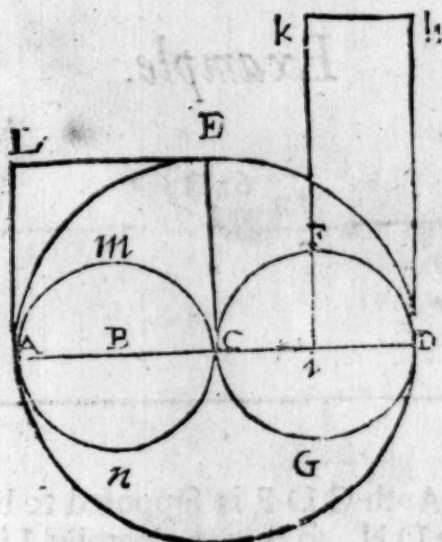
All

All Artifts allow, that the multiplying half the Circumference by half the Diameter, produces the superficial Content in Inches, but why it does so is left to the Ingenuous: We shall suppose the Diameter of this Circle to be 40 Inches, and Circumference to be 125.6, the half of 40 is 20, and the half of 125.6 is 62.8, the Product of these two multiplied into each other is 1256.0, the Content of the Circle.

Quest. What is the Reason you multiply half the Diameter by half the Circumference, and does not multiply the whole of the one by the whole of the other, to find the superficial Content?

Answ. If you should multiply the whole Circumference by the whole Diameter, it would produce a Square twice as large as the Circle; for a Circle laid down with the extension of the Compasses from A to C, as in the following Diagram appeareth, will be four times as large as one laid down with an extension from B to C.

Proof



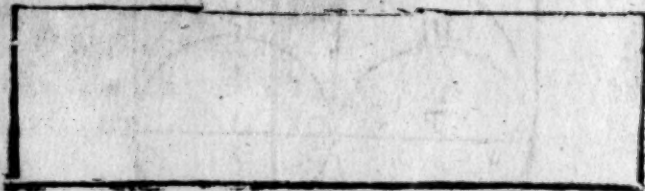
Proof. Suppose the Diameter of the Circle ABC be 40 Inches, and Circumference 125.6 Inches, the Product of the half of the one multiplied into the half of the other is 1256.0 Inches, which is equal to the Quadrant ACE of the large Circle, and is less by almost $\frac{1}{4}$ than the Square made of the Diameter of ABC.

Now suppose the Semi-diameter of CDFG to be 20, the half of the Circumference CFD to be 62.8, if you lay down an Oblong, whose longest side is 62.8, and shortest side 20, the square Inches therein will be equal to 1256.0, the Content of the Circle CDFG.

Example.

62.8

20



For the Arch CDF is supposed to be extended as the Line DH , so that the parallel Lines IK and KH makes up the Oblong; for as HD is equal to CDF , so is IK equal to CGD , and so is ID to its parallel equal.

Now, if I were to measure the Content of this Oblong $IDHK$, (which is supposed equal to the Circle $CFDG$) the vulgar way to perform it is to multiply one of the long sides by one of the shorter, as IK by ID , but ID is supposed equal to the Radius or Semi-diameter of the Circle, and IK equal to the half of the Circumference; so that if in the Mensuration of Circles we went about to multiply the whole Diameter by the whole Circumference, it would be as absurd, as if, in order to find out the Content of a Square, we multiplied all the four sides into one another, for what Proportion two sides of a Square have to the whole Square, the same Proportion have the half of the Circumference, and the half of the Diameter to the whole Circle.

Quest.

Quest. What is the Reason that you square the mean Diameter of a Circle, and multiplies the Square by 11, and divides the Product by 14, to find the Content?

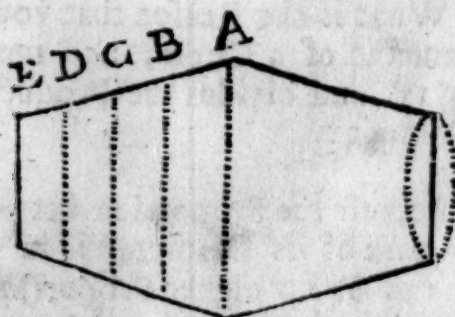
Ans. Because the Proportion between a Circle and the Square of its Diameter is as 11 is to 14, for there can be no higher Proportion found (in integral Numbers) a Geometrical Square, as LECA, whose sides are all equal to the Diameter of the Circle A M C N, will be greater than the Circle by ALE. Therefore the difference between a Square and a Circle corresponds to the difference betwixt 11 and 14.

The foregoing Demonstrations are enough, if well observed, to make a Man able to Gauge any Vessel, for the whole Art of Gauging is grounded thereupon; but I shall now proceed to the use of Circles in Cask Gauging.

Of Cask Gauging.

LET a Cask be of what form soever, it must be reduced into a Cylinder, that is, a Solid, whose Circumference is equal in all parts; as suppose it were a Cask in form as the Figure following, viz. The Frustum of two Cones abutting upon a common Base.

All



All the way between the Bung and Head is supposed to be nothing but Circles, as if it were so many points as could be set down touching each other, which at last makes an individual Line; for as a point is supposed indivisible, a number of them touching each other makes an entire Line.

Now having the Diameter at A and likewise at E, take half the Sum, and you have the Diameter at C, so likewise if you have the Diameters at C and E, the half of the Sum is the Diameter at D; so that C is certainly the mean Diameter of ABCDE, and consequently the Cask is reduced into a mean Circle or Cylinder, the which being squared, and the Square divided by the proper Divisor for Ale or Wine, you have the Area, or what it holds upon one Inch.

Quest. Why is 359.105 a proper Divisor to divide the Square of a Diameter by, to find the Area for Ale?

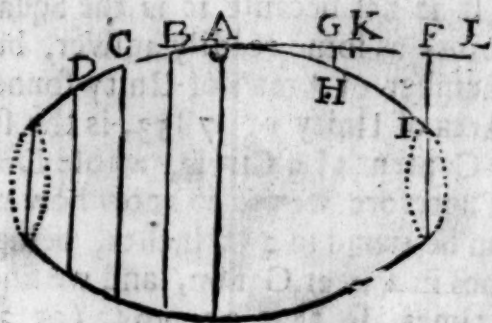
Ansiv.

Answ. It is not because it is the Square of the Gauge point, as some readily answer, but because it is the number of Area's of Unity found in 282. For the Area of Unity or .7853, is the superficial or square Content of a Circle, whose Diameter is 1 Inch. Therefore we are to know how many such Area's can be found in 282 Inches, being the Cubical Inches in a Beer Gallon, and we find it to be 359.105 times, so that 359.105, (or 359 only) is proper for a Circle, as 282 is for a Square, for if 282 be the number of square Inches in a Gallon, 359.105 is the number of Circles, whose Diameter is one Inch, contained in a Gallon of Beer or Ale.

*Of Gauging the middle Frustum
of a Spheroid.*

AS I have set forth in that of Conoids before mentioned, you are to conceive this Cask to be so many Circles, as indivisible points may stand between End and End, but by reason that from Bung to Head either way is equal, we shall only suppose it to be the double of one end.

Now



Now suppose your Diameter at A to be 35, the Diameter at B must be less, for according to *Enclid*, Lib. 3. Prop. 13. a lesser Circle within a greater can touch the greater but in one point, therefore you are to suppose the Line A E reduces the Cask to the Frustums of Conoids, and the Segment A B C D E to be found according to the Problems of Geometry too large here to insert, my aim here being only to shew the Basis and reasonable part of Gauging.

Quest. What is the Reason that the Sum of the double Square of the Bung and Square of the Head multiplied by the length and divided by 1077. gives the Content of the Frustum of a Spheroid, as is directed by several Authors?

Ans. Because the difference between F and G is twice as much as between G and A, for if the Arch A H be extended, it will reach to K, but if the Arch H I be extended it will reach to L, which shews that the difference between L and F is twice as much as that between K G, so consequently you must take two Circles at the Bung, to one at the Head

Head to reduce it to a Cylinder, but the exactest way to reduce it and find a Mean is, to take the Diameter at every Inch, and divide the Sum by the number of Inches between Head and Bung, and the Quotient will be very nigh the mean Diameter, but the best Artist can never reduce it to a Cylinder.

Quest. Why do you make use of 1077 for a Divisor in such a Case to find the Content?

Ans. It may be easily answered, *Because it is triple the Square of the Gauge-point,* but was I to receive such an Answer I should not take it as a sufficient Resolution, but should demand why it is used in such a Case, tho' it is the triple Square thereof.

Now the double Square of the Bung Diameter and the Square of the Head Diameter being added together, makes the Sum of the Square of three Circles, therefore you are to divide the Sum of these three Squares by thrice the Square of the Gauge-point, for if you was to divide that Sum by 359, being the proper Divisor for the Square of one Circle, it would Quote thrice as much as the Cask holds.

As for Example. We shall suppose a Cask in form before mentioned to have such Dimensions as follows, *viz.*

Length 42	} Double Square of the Bung and the Square of the Head is	} 3626.63
Bung 36.3		
Head 31.5		

C

Divide

Divide this Sum by 1077, and it quotes such a Number as when multiplied by the Casks length is equal to the Content of the Cask, whereas if you divide by 359, it will produce three times the Content.

1077)3626.63(3.367

3956

7253

7910

371

This Quotient multiplied by the Casks length produces 121.414 Gallons, which is the true Content of the Cask in Ale Gallons.

Now I shall divide the same Number, viz. the Sum of double the Square of the Bung Diameter, and the single Square of the Head Diameter, by 359, and it will bring forth a Number which will be equal to thrice the Content of the Cask.

Example.

359)3626.63(10.102

366

42

730

20204

12

40408

424.284

The

The foregoing Work is clear enough to the Intelligible why 1077 is a proper Divisor, and if that Reason be duly considered it will be of great use in Gauging Vessels of other forms.

Since there is no Proportion found between a Square and a Circle, I can't see any Reason why we should take it for granted, that the Square of the mean Diameter, divided by 359, should produce the Area, and why we may not use 282 as a Divisor for Circles as well as for Squares, when the best Authors allow that the Area of a Circle is found by multiplying half the Diameter by half the Circumference.

Now if you have a Cask whose Diameter is 80, its Circumference must be 251.20, the half of these two being multiplied into each other, and divided by 282, will produce 17.8 Gallons, for by the Rule of Proportion, as 1 is to the Product of the half of the Diameter, and half of the Circumference, multiplied into each other, so is 282 to the Area of the Cask in Ale Gallons.

Example.

$$1 : 5024 : 282 (17.8$$

I am of Opinion there may be a more exact way found to measure curved Lines than any now used, if the Angle of Contact can be found commensurable, for then it might be easie to find a straight Line equal to a curved Line, and by consequence a

Square equal to a Circle, but the Angle of Contact being (as *Clavius* says) a Quantity greater than no Quantity, and the least of all Quantities, there may be an allowance made for it, according to the length of the Cask. More of this another time.

I shall conclude this small Work, hoping that whoever peruses it, and after Consideration shall alledge it erroneous, he will be so generous, as not to say, *It is false, because it is so*, using the Woman's Reason, but demonstratively prove it so by Geometry, and after a reasonable Confutation, I shall not only own my self obliged to him, but for the future shall pray his Approbation to whatever I do in that nature. *So Farewel.*

Now if you have a Cask whose Diameter is 20,
its Circumference may be 251.20, the half of this
being multiplied into each other, and divided
by 25, will produce 178 Gallons, for by the Rule
of Proportion, as 1 is to the product of the half of
the Diameter, and half of the Circumference, thus,
1 : 251.20 :: 178 : the Area of
the Cask is 178 Gallons.

FINIS.

1 : 251.20 :: 178 : 8

I am of Opinion there may be a more exact way
found to measure curved Lines than any now used,
if the Angle of Contact be to be considered as a straight
Line, for then it is equal to a straight Line, and by consequence
the Area equal to a straight Line, and by consequence
square

29 JA 66

